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Tune calculation at the Injection

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Abstract

Injection tunes in the Booster can be adjusted *via* quadrupole correctors. Quadrupole correctors at short and long sections are ramped as two different groups, and they control horizontal and vertical tunes separately since the horizontal beta function reaches the maximum at short sections and the vertical beta function reaches the maximum at long sections. A simple linear model is used to calculate the tune based upon quad corrector settings.

Method

Quads at short and long sections are represented by *QS* and *QL*, and their currents are I_{tunes} and I_{tunel}. Horizontal and vertical tunes are ν_x and ν_y. Equations (1) and (2) are used to calculate ν_x and ν_y once a, b, c, d, e, and f are known.

$$\nu_x = a \cdot I_{tunes} + b \cdot I_{tunel} + e \qquad (1)$$

$$\nu_y = c \cdot I_{tunes} + d \cdot I_{tunel} + f \qquad (2)$$

Coefficients a, b, c, d, e, and f are to be determined, and first is to find a and c. I_{tunel} is fixed at 0.54 A, and I_{tunes} is varied from -0.35 A to 0.55 A with a step of 0.1 A; ν_x and ν_y are calculated at each set of I_{tunes} and I_{tunel} using MAD. Afterwards, equations (3) and (4) are used to calculate a and c, and results are "a = 0.43735" and "c = -0.07385".

$$\overline{\Delta\nu_x} = a \cdot \Delta I_{tunes} \qquad (3)$$

$$\overline{\Delta\nu_y} = c \cdot \Delta I_{tunes} \qquad (4)$$

$$\Delta I_{tunes} = 0.1\,\mathrm{A}, \qquad \overline{\Delta\nu_x} = \sum_{i=1}^{9} (\Delta\nu_x)_i \Big/ 9, \quad \overline{\Delta\nu_y} = \sum_{i=1}^{9} (\Delta\nu_y)_i \Big/ 9$$

Second, b and d are to be determined. I_{tunes} is fixed at -0.2178 A, and I_{tunel} is varied from 0.04 A to 0.54 A with a step of 0.1 A; ν_x and ν_y are calculated at each set of I_{tunes} and I_{tunel} using MAD. Afterwards, equations (5) and (6) are used to calculate b and d, and results are "b = 0.08529" and "d = -0.2401".

$$\overline{\Delta\nu_x} = b \cdot \Delta I_{tunel} \qquad (5)$$

$$\overline{\Delta\nu_y} = d \cdot \Delta I_{tunel} \qquad (6)$$

$$\Delta I_{tunel} = 0.1\,\mathrm{A}, \qquad \overline{\Delta\nu_x} = \sum_{i=1}^{5} (\Delta\nu_x)_i \Big/ 5, \quad \overline{\Delta\nu_y} = \sum_{i=1}^{5} (\Delta\nu_y)_i \Big/ 5$$

Finally, e and f are determined using equations (1) and (2) since a, b, c, and d are already known, and results are "e = 6.70475" and "f = 6.83989".

We can calculate the lattice tune at injection using equation (7) and (8) with the input of quad corrector settings, I_{tunes} and I_{tunel}.

$$\nu_x = 0.43735 \cdot I_{tunes} + 0.08529 \cdot I_{tunel} + 6.70475 \qquad (7)$$

$$\nu_y = -0.07385 \cdot I_{tunes} - 0.2401 \cdot I_{tunel} + 6.8400 \qquad (8)$$

Check the result

The tune shift due to quad correctors can be estimated using equation (9).[1]

$$\delta\nu_{x,y} = \frac{1}{4\pi} \sum_{\mathrm{i}} \frac{(\beta_{x,y})_{\mathrm{i}}}{f_i} \qquad (9)$$

When "I_{tunel} = 0.0 A" and "I_{tunes} = 0.0 A", "ν_x = 6.704553" is obtained using MAD. "$\beta_{\mathrm{x}} \approx 33.0$ m" at short sections, "$\beta_{\mathrm{x}} \approx 7.0$ m" at long sections, and f^{-1} can be calculated using equation (10).[2]

$$1/\mathrm{f} = 0.3 \cdot k_{quad} \cdot I_{tune} \cdot l / (P_c \cdot l) \qquad (10)$$

$$k_{quad} = 0.02178\ \mathrm{T/m},\ P_c = 0.954263\ \mathrm{GeV/c\ at\ injection}$$

Here, f can be f_s or f_l, and I_{tune} can be I_{tunes} or I_{tunel}.

When "I_{tunel} = 0.54 A" and "I_{tunes} =0.35 A", $\delta\nu_x$ is about 0.151 from quads at short sections, and it's about 0.05 from quads at long sections, so ν_x is estimated to be 6.9055(=6.70455+0.151+0.05) using equation (9).

Using the linear model, we get $\nu_x = 0.43735 \cdot 0.35 + 0.08529 \cdot 0.54 + 6.70475 = 6.9039$, and it's very close to the result which is estimated using equation (9).

Conclusion

The injection tune calculated by the linear model has been checked using the analytical formula, and there is a good agreement. The linear model for the tune calculation can be applied to the operation or the tune scan at injection.

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References:

[1] D. A. Edwards and M. J. Syphers, An Introduction to the Physics of High Energy Accelerators.

[2] A. Drozhdin, Booster MAD input file.